SRR & CVR GOVT. DEGREE COLLEGE

(Autonomous) NAAC ‘B^{+}’ Grade

DEPARTMENT OF MATHEMATICS

II B.Sc. MATHEMATICS

SEMESTER - IV, PAPER - V

LINEAR ALGEBRA

MODEL QUESTION PAPER

Time: 3 Hrs **Max. Marks : 60**

SECTION - A

Answer any *FIVE* questions. **5 X 4 = 20 M**

1. Let p, q, r be the fixed elements of a field F. Show that the set W of all triads (x, y, z) of elements of F such that $px + qy + rz = 0$ is a subspace of $V_3(F)$.
2. If S is a subset of a vector space V(F), then prove that
   (a) S is a subspace of $V \Leftrightarrow L(S) = S$ and (b) L(L(S) = L(S).
3. If $\alpha,\ \beta,\ \gamma$ are linearly independent vectors of a vector space V(F), then show that $\alpha + \beta,\ \beta + \gamma,\ \gamma + a$ are also linearly independent.
4. Show that the set $\{(1, 0, 0),\ (1, 1, 0),\ (1, 1, 1)\}$ is a basis of $C^3(C)$ and hence find the coordinates of the vector $(3 + 4i,\ 6i,\ 3 + 7i)$ in $C^3(C)$.
5. If the mapping $T : V_3(R) \to V_2(R)$ is defined by $T(x,\ y,\ z) = (x - y,\ x + z)$ then show that T is a linear transformation.
6. If $T : V_3(R) \to V_3(R)$ is a linear transformation defined by T(a, b, c) = $(3a,\ a - b,$ $2a+b+c$, then show that $T2-IT2-3I=0$.
7. Find the characteristic roots of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.
8. Prove that the characteristic vectors corresponding to the distinct characteristic roots of a matrix are linearly independent.
9. Find the unit vector orthogonal to $(4,\ 2,\ 3)$ in R^3.
10. State and prove Triangle inequality.

SECTION – B

Answer *ALL* the following questions. **5 X 8 = 40 M**

11. (a). Show that the necessary and sufficient condition for a non empty subset W of a vector space V(F) to be a subspace of V is that $a, b \in F, \alpha, \beta \epsilon W \Longrightarrow a\alpha + b\beta \in W$.

(Or)

(b). If W_1 and W_2 are subspaces of a vector space V(F) then show that $W_1 + W_2$ is a subspace of V(F) and $L(w_1 \cup w_2) = w_1 + w_{2.}$

12. (a). Let W_1 and W_2 be two subspaces of R^4 given by $W_1 = \{(a, b, c, d) : b - 2c + d = 0\}$ and $W_2 = \{(a, b, c, d) : a = db = 2c\}$. Find the basis and dimension of (i) W_1, (ii) W_2, (iii) $W_1 \cap W_2$ and hence find (iv) $\dim(W_1 + W_2)$.

(Or)

(b). Let V(F) be a finite dimension vector space and $S = \{\alpha_1, \alpha_2, \ldots, \alpha_3\}$ is L.I. subset of V. Then prove that either S itself a basis of V or S can be extended to form a basis of V.

13. (a). State and prove Rank – Nullity theorem.

(Or)

(b). Find the null space, range, rank and nullity of the transformation $T : R^2 \to R^3$ defined by $T(x, y) = (x + y,\ x - y,\ y)$.

14. (a). State and prove Cayley – Hamilton theorem.

(Or)

(b). Solve $x + y + z = 6,\ x - y + z = 2,\ 2x - y + 3z = 9$ by matrix inversion method.

15. (a). State and prove Cauchy – Schwarz's inequality.

(Or)

(b). Applying Gram – Schmidt orthogonalisation process obtain an orthonormal basis of R^3 from the basis $\{(1, 0, 1),\ (1,\ 0,\ -1),\ (0,\ 3,\ 4)\}$.